ISSN 2229-5518

MEAN, VARIANCE AND STANDARD DEVIATION TO THE EMPIRICAL CONTINUOUS DISTRIBUTION FUNCTION ON DEPARTURES

Abstract: This paper proposes the Mean, Variance and Standard Deviation for density function

$$f(x)=\begin{cases}\dfrac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!} & for\ \ n\geq 0\\ 0 & otherwise\end{cases}$$

for the chosen random variable "departing only (N-n) persons from the system".

Key Words: Random variable; Continuous probability distribution; Density function; Mean; Variance; Standard Deviation.

Introduction Due to its universality, Statistics has the most interesting solutions for the problems in several fields. By taking some inconsiderable subtle transformations on the existing distributions several new distributions have been developed. This paper is continuous work to the previous work [2] which briefed here. The Random variable of interest is to "departing only (N-n) persons from the system". Instead of asking "In fixed time how many departures will take place?", We ask how likely the interval to have 'N-n' fixed no. of departures out of N arrivals. Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is some distribution of X. We charted the histograms for different number of departures from the system from which we found the density curves.

In which case its probability density function is given by

$$f(x)=\begin{cases}\dfrac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!} & for\ \ n\geq 0\\ 0 & otherwise\end{cases}$$

ISSN 2229-5518

where N is the restricted number of arrivals to the system.

μ Is the departure rate.

n is no. of persons remained in the system after taking (N-n) persons their service.

Graph of density function:

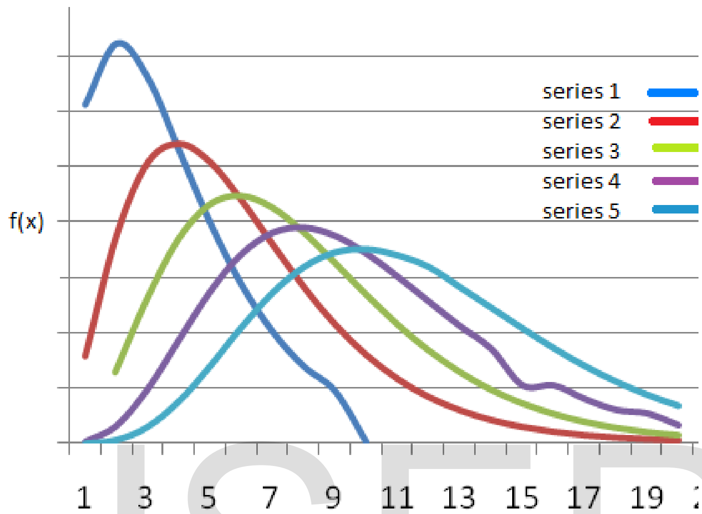

Figure - 1

Series 1 represents the probability density function graph of new random variable when n = 0.

Series 2 represents the probability density function graph of new random variable when n = 1.

Series 3 represents the probability density function graph of new random variable when n = 2.

Series 4 represents the probability density function graph of new random variable when n = 3.

Series 5 represents the probability density function graph of new random variable when n = 4.

X- Axis represents time; Y- represents f(x).

We extend this distribution theory, to calculate the interesting properties of all statistical distributions, mean, variance and the standard deviation. These are studied due to their wide use in several fields like management, insurance, business and finance etc.

MEAN, VARIANCE AND STANDARD DEVIATION OF THE PROBABILITY DISTRIBUTION

The probability density function is

$$f(x)=\begin{cases}\dfrac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!} & for\ \ n\geq 0\\ 0 \quad otherwise\end{cases}$$

where N is the restricted number of arrivals to the system.

μ Is the departure rate.

n is no. of persons remained in the system after taking (N-n) persons their service.

For N-n=0 i.e. n= N

Mean $E(x)=\int_0^\infty xf(x)dx$

Here $f(x)=\dfrac{(\mu x)^{N-n}\mu e^{-\mu x}}{(N-n)^1}$

Mean $\int xe^{-\mu x}\mu dx=\dfrac{1}{\mu}$

Variance $\int x^2e^{-\mu x}\mu dx-(E(x))^2 = \dfrac{1}{\mu^2}$

For n= N-1, N-n=1

Mean = E(x) = $\int_0^\infty xf(x)dx$

$=\int_0^\infty xe^{-\mu x}(\mu x)^1\mu dx=\dfrac{2}{\mu}$

Variance $=\int_0^\infty x^2e^{-\mu x}(\mu x)^1\mu=\dfrac{2}{\mu^2}$

For n = N-2, N-n = 2

Mean $E(x) \ =\int_0^\infty xf(x)dx$

$$\int_0^\infty \frac{xe^{-\mu x}\left(\mu x\right)\mu}{2!}dx = \frac{3}{\mu}$$

Variance $V(x) = \int_0^\infty x^2 \frac{\mu(\mu x)^2 e^{-\mu x}}{2!}dx = \frac{3}{\mu^2}$

For n=N-3, N-n=3

Mean = $= \int_0^\infty \frac{x\mu\left(\mu x\right)^3 e^{-\mu x}}{3!} = \frac{4}{\mu}$

Variance $\int_0^\infty \frac{x^2\mu\left(\mu x\right)^3 e^{-\mu x}}{3!} = \frac{4}{\mu^2}$

	n=N-0	n=N-1	n=N-2	n=N-3	n=N-N
	N-n=0	N-n=1	N-n=2	N-n=3	N-n= N
Mean	$\frac{1}{\mu}$	$\frac{2}{\mu}$	$\frac{3}{\mu}$	$\frac{4}{\mu}$	$\frac{N+1}{\mu}$
Variance	$\frac{1}{\mu^2}$	$\frac{2}{\mu^2}$	$\frac{3}{\mu^2}$	$\frac{4}{\mu}$	$\frac{N+1}{\mu^2}$
Standard Deviation	$\frac{1}{\mu}$	$\frac{\sqrt{2}}{\mu}$	$\frac{\sqrt{3}}{\mu}$	$\frac{2}{\mu}$	$\frac{\sqrt{N+1}}{\mu}$

In general we write Mean of the above probability distribution is E(x) = $\frac{n+1}{\mu}$

Variance of the above probability distribution is $V\left(x\right) = \frac{n+1}{\mu^2}$

Standard deviation of the above distribution is $\frac{\sqrt{n+1}}{\mu}$

3.6 CONCLUSION

In this paper for the Truncated Probability density function of departures, Mean, variance and standard deviation is calculated.

PART-1[2]

For the assumed random variable, the truncated density function proposed is

$$f(x) = \begin{cases} \dfrac{e^{-\mu x}(ux)^{N-n}\mu}{(N-n)!} & for\ x \geq 0 \\ 0 & othewise \end{cases}$$

PART-2 In general Mean of the above probability distribution is E(x) = $\dfrac{n+1}{\mu}$

Variance of the above probability distribution is $V(x) = \dfrac{n+1}{\mu^2}$

Standard deviation of the above distribution is $\dfrac{\sqrt{n+1}}{\mu}$

REFERENCES

[1] Daisuke Nakezato, Demitriics Bertsimas The distributional Little's law and its applications, (with D.Nakazato), *Operations Research*, 43, 2, 298-310, MIT, 1995.

[2] Nirmala Kasturi, A Continuous Probability Distribution on Departures; Article Accepted by IJERD Journal. Ref. ID AB712013.

[3] Hamdy Taha: Operations research An Introduction, seventh edition, pearson education.

[4] James M Thomson and Donald Gross; Fundamentals of Queueing theory, Third addition, 2002: Wiley series.

[5] Janus Sztrik, Queueing theory and its applications; A personal view by Proceedings of the 8th International conference on Applied Informatics, Eger, Hungary, Jan 27-30, 2010, Vol.1, PP.9-30.

[6] Jeffrey Herrman: A survey of Queueing theory Applications in Healthcare, Samue Fomundam ISR Technical Report 2007-24.

[7] J-P Cletch, D.M.Noctor, J.C.Manock, G.W.Lynott and F.E.Bader; Surface Mount Assembly failure statistics and failure free items AT&T Bell laboratories, 44th ECTC, Washington, D.C, May 1-4, 1994, PP.293-297.

[8] Michael D. Peterson, Dimitris J. Bertsimas, Amedeo R. Odoni; MIT, Models and algorithms for Transient Queueing Congestion at Airports, Management science, volume 41, 1279-1295, 1995.

[9] Demitriics BertsimasAn analytic approach to a general class of G/G/s queuing systems, *Operations Research*, 38, 1, 139-155, MIT, 1990.

[10] S.D. Sharma; Operations research, Fourteenth addition, 2003, Kedarnadh and Ramnadh and co. publishers.

[11] W. Weibull "A statistical distribution function of wide applicability" Journal of applied Mechanics, September 1951, pp 293-297.

[12] R.Luus and Jammer; Estimation of parameters in 3 parameter Weibull probability distribution functions, 10th international workshop on Chemical Engineering Mathematics, Budapet, Hungary, August 18-20 (2005).

[13] Patric O'Conner, Reliability Engineering, 5th Edition, 2012, Wiley Series.

[14] William W. Hines, Douglas C. Montgomerry, David M.Goldsman, Connie M. Borrer, probability and Statistics in Engineering, Fourth Edition, Wiley Series.

[15] Vijay K. Rohatgi, A.K.Md. Ehsanes Saleh, An Introduction to Probability And Statistics, Second Edition, Wiley Series.

[16] Kishore S. Trivedi, Probability & Statistics with Reliability, Queueing And Computer Science Applications, Second Edition, Wiley Series.

[17] Ravindran, Phillips, Solberg, Operations Research principles and practice, Second Edition, Wiley series.

[18] R.Sree parimala, S.Palaniammal Bulk Service Queueing Model With Services Single and Delayed Vacation, International Journal of Advances in Science and Technology(IJAST), Vol 2, Issue 2, June 2014.

[19] J. D. C. Little, "A Proof for the Queuing Formula: Operations Research, vol. 9(3), 1961, pp. 383-387, doi: 10.2307/167570.

[20] M. Laguna and J. Marklund, Business Process Modeling, Simulation and Dsign, Pearson Prentice Hall, 2005.

[21] Gamarnik, On the Undecidability of Computing Stationary Distributions and Large Deviation Rates for Constrained Random Walks, Mathematics of Operations Research, 2007, Vol. 32, 257-265.

[22] Gamarnik, Katz, On Deciding Stability of Multiclass Queueing Networks under Buffer Priority Scheduling Policies, Mathematics of Operations Research, 2009, Vol. 19, 2008-2037.

[23] Ajeet Kumar Pandey, Srinivas Panchangam, Jessy George Smith, Safety analysis of Automatic Door operation for Metro Train : A case study. 9th International conference on Heterogeneous Networking for quality, reliability Robustness, January 11-12-2013, Greater Noida, India.

[24] Wallace Agiel, Christian Asave, Modeling and Analysis of Queueing Systems in Banks, International Journal of Scientific Technology, Volume 4, Issue 7, July 2015.

[25] T.L. Pap and L. Leiszner, The use of Queueing Theory for planning automated analytical systems, Journal of Automated Chemistry, volume 9, No. 2 (April- June 1987), p.p 87-91.

[26] X. Papaconstantinou, On the exact steady state solution of the Ek/C2/s queue, *European Journal of Operations Research*, 37(2), 272-287, 1988.

[27] J Keilson, D, Nakazato, H, Transient and busy period analysis of the GI/G/1 queue as a Hilbert factorization problem, *Journal of Applied Probability*, 28, 873-885, 1991.